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THE AFFINE CLASSIFICATION OF CUBIC CURVES

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ABSTRACT. The orbits of the action of the affine group of $\mathbf{R}^2$ on the real plane cubic curves given by $Ax^3 + Bx^2y + Cxy^2 + Dy^3 + Ex^2 + Fxy + Gy^2 + Hx + Iy + J = 0$ are computed.

Introduction. Since the pioneering work of Isaac Newton, there have been many classifications of cubic curves, based on a variety of criteria. The turning point in the approach to the classification problem came with the Klein Erlangen Program, which places geometry on a group theoretic foundation. Until that time the criteria used can best be described as non-group theoretic and were primarily due to Newton, Plucker, and Cayley. (See [**1, 3, 7, 9**].) For classifications that fall within the scope of the Klein Erlangen Program see [**2, 4, 6, 11, 12**]. The classification in [**2**] is cased on the Euclidean motion group of the plane, while that in [**4, 6, 11,**] or [**12**] is based on the complex projective group. Recent classifications by equisingularity due to C.T.C. Wall combine certain aspects of both major approaches. (See [**13** and **14**].)

In this paper, a Kleinian approach based on the affine group of $\mathbf{R}^2$ is presented. The purpose of this paper is to compute the orbits of the action of the affine group of $\mathbf{R}^2$ on the real plane cubic curves and to exhibit a complete set of equivalence class representatives. Formulated in an algebraic way, the central question of this paper is: (Q) How much can the equation $Ax^3+Bx^2y+Cxy^2+Dy^3+Ex^2+Fxy+Gy^2+Hx+Iy+J=0$ be simplified by performing the following two types of operations:

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