

PRODUCT COMPLEX SUBMANIFOLD OF INDEFINITE COMPLEX SPACE FORMS

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ABSTRACT. We study product complex submanifolds of indefinite complex space forms and characterize a submanifold which corresponds to the Segre imbedding in a definite complex space form.

0. Introduction. It is well-known that for complex hyperbolic spaces an analogue to the Segre imbedding cannot be given. The key to this is that if there is a holomorphic isometric immersion of a product of two Kaehler manifolds into a complex space form, then the holomorphic sectional curvature of the ambient space has to be non-negative (see [8]). A careful observation to the proof of this fact tells us that the positivity of the metric in each normal space to the submanifold also must be taken into account. Thus the following problems arise in a natural way:

1. Does a product of two complex hyperbolic spaces admit a holomorphic isometric imbedding in an “indefinite” complex hyperbolic space?
2. If the answer is yes, find the smallest possible dimension and index of such an indefinite complex hyperbolic space.
3. Characterize this holomorphic isometric imbedding when dimension and index are as small as possible.

In this paper these problems will be solved in a more general context. In §1, an indefinite analogue to the Segre imbedding for indefinite complex projective (and hyperbolic) spaces is given. Using the relationship between indefinite complex hyperbolic spaces and indefinite complex projective spaces, we obtain a holomorphic isometric imbedding of a product of definite complex hyperbolic spaces into an “indefinite” complex hyperbolic space. In §2, some basic formulas are recalled for later use. In §3, an answer to the second problem is given (Corollary 3.2)

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