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A RANGE PROBLEM FOR HOMOGENEOUS, HYPERBOLIC PARTIAL DIFFERENTIAL EQUATIONS

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1. Introduction A range problem will be considered for the partial differential equation $Pu = f$ where $u(x;t)$ and $f(x;t)$ are real-valued functions on $\mathbf{R}^n \times \mathbf{R}$ and $P = P\left(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}, \frac{\partial}{\partial t}\right)$ is linear, homogeneous with constant coefficients, and hyperbolic with respect to t. The question to be considered is: if f is supported in the bounded set Ω and u is known for values of t such that $\mathbf{R}^n \times \{t\}$ is disjoint from Ω, can f be determined? For simplicity, it will be assumed herein that u vanishes when $t <$ inf $\{\tau : \Omega \cap (\mathbf{R}^n \times \{\tau\}) \neq \emptyset\}$. For a physical system modeled by the classical wave equation, this question is equivalent to asking if a force of finite extent and duration can be found from the subsequent disturbance that it generates.

One elementary observation regarding this question can be made immediately. Whereas u can be found from f by classical, explicit formulas, f is not uniquely determined by the values of u outside Ω. Indeed, for v also supported in $\Omega, f + Pv$ yields a solution which coincides with those values of u.

The main result of this paper is the following theorem. $L_0^2(\Omega)$ will denote the square-integrable, real-valued functions having support in Ω.

THEOREM 1.1. *Let Ω be an open, convex, and bounded subset of $\mathbf{R}^n \times \mathbf{R}, f \in L_0^2(\Omega)$, and $P = P\left(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}, \frac{\partial}{\partial t}\right)$ be linear with constant coefficients, homogeneous, and hyperbolic with respect to t. Suppose $u(x;t)$ vanishes for large negative values of t and satisfies $Pu = f$. Then, for $t > T =$ sup $\{\tau : \Omega \cap (\mathbf{R}^n \times \{\tau\}) \neq \emptyset\}$, a representative of the class $[f] = \{f + Pv : v, Pv \in L_0^2(\Omega)\}$ can be computed from the Cauchy data for u on $\mathbf{R}^n \times \{t\}$. Furthermore,*

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