ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 18, Number 2, Spring 1988

NEWTON FLOWS FOR REAL EQUATIONS

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1. Introduction. Let $G : \mathbf{R}^n \to \mathbf{R}^n$ be a smooth mapping with Jacobian matrix $DG(x)$. In this paper we shall discuss the dynamical system

$$N(x) = x - DG(x)^{-1}G(x) \tag{1}$$

provided by Newton's method for the system of equations

$$G(x) = 0. \tag{2}$$

If $n = 2$ and G is a rational mapping R of the complex plane $\mathbf{C}$, then the dynamics of (1), though possibly very complicated and delicate, is understood in terms of the classical and recent theory of *Julia sets* [**3, 4, 1**]. In particular, since ∞ is typically a repelling fixed point of N one has that

$$J_N = \text{ closure } \{x \in \overline{\mathbf{C}} : N^k(x) = \infty, \text{ for some } k \in \mathbf{N}\} \tag{3}$$

is the Julia set of $N(x) = x - R(x)/R'(x)$ (here $\overline{\mathbf{C}} = \mathbf{C} \cup \{\infty\}$ and $N^k = N \circ \cdots \circ N$ k-times). Moreover, if $\overline{x} \in \mathbf{C}$ is a simple zero of R, i.e., $R'(\overline{x}) \neq 0$, then $\overline{x}$ is an attractive fixed point of N; if

$$A(\overline{x}) = \{x \in \mathbf{C} : N^k(x) \to \overline{x} \text{ as } k \to \infty\}, \tag{4}$$

is its basin of attraction, then

$$\partial A(\overline{x}) = J_N. \tag{5}$$

Since (5) is true for any attractive fixed point of N (or even cycles), J_N is typically a *fractal* set which in addition has the interesting property that Newton's method clearly will diverge for initial values in J_N. On the other hand, if n is not restricted to be 1 or 2 and G is simply

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Supported by Stiftung Volkswagenwerk and by NSF grant DMS-8501311.
Received by the editors on May 14, 1986.