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ON THE COLLAPSE OF THE RESONANCE STRUCTURE IN A THREE-PARAMETER FAMILY OF COUPLED OSCILLATORS

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1. Introduction. In many biological systems it is necessary to scynchronize or otherwise organize the temporal activity of a population of cells, and this is usually achieved through stimulation or 'forcing' by a pacemaker, or by mutual coupling within the population. While the pacemaker is sometimes external to the organism, as in the case of circadian rhythms, many interesting examples in physiology involve endogenous pacemakers. Examples include the oscillatory networks of neurons (the central pattern generators) that underlie a variety of periodic behavior [**1, 2**] and the SA node in the mammalian heart. In the SA node individual cells generate periodic outputs, and the problem is to understand how the output is synchronized in the population [**3**]. In central pattern generators the periodic output is often a network property, in that individual cells do not burst periodically in isolation [**1**], and the problem is to understand the patterns of interaction that can generate the observed periodic behavior. However, synchronization is not always desirable, as is illustrated by the fact that synchronized bursting of large numbers of neurons underlies epileptic seizures [**4**]. Many other examples can be given to underscore the fact that knowledge of how coupling affects the collective behavior of aggregates of cells is important for understanding both normal and pathological processes in numerous biological and physiological systems [**5**].

From a mathematical standpoint the simplest system that is relevant in this context is a single periodically-forced system, and for such systems much is known about the dependence of solutions on the period and amplitude of the forcing function [**2**]. However, much less is known about the dependence of solutions for a system of coupled oscillators on biologically-relevant parameters, such as the intrinsic frequency of the oscillators and the coupling strength. Some results can be gotten by asymptotic methods in the limit of very weak or very

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