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MULTI-GRID CONTINUATION AND SPURIOUS SOLUTIONS FOR NONLINEAR BOUNDARY VALUE PROBLEMS

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ABSTRACT. Recently the author together with R. Bank has developed, implemented and successfully applied a continuation technique for the numerical solution of parameter-dependent nonlinear elliptic boundary value problems. The method was integrated into an existing multi-grid package based on an adaptive finite element discretization. We present the continuation method and prove an important theoretical result for the corrector iteration. For the Bratu problem $-\Delta u = \lambda e^u$ on the square with homogeneous Dirichlet conditions we show how spurious solutions maybe encountered while computing relevant solutions, how the program handles those and how it allows to detect them.

1. Introduction. In the following we consider the parameter-dependent nonlinear problem

$$G(u,\alpha) = 0, \tag{1.1}$$

where $G : X^m \times \mathbf{R}^p \to X^m, X$ a suitable function space and α a vector of real parameters. (1.1) will typically represent a parameter-dependent nonlinear elliptic system of dimension m and order two. Important examples are the VLSI device simulation equations

$$\begin{aligned} g_1(u,v,w) &\equiv -\Delta u + e^{u-v} - e^{w-u} - k_1 = 0, \\ g_2(u,v,w) &\equiv \nabla \cdot \mu_n e^{u-v} \nabla v + k_2 = 0, \\ g_3(u,v,w) &\equiv -\nabla \cdot \mu_p e^{w-u} \nabla w + k_2 = 0. \end{aligned} \tag{1.2}$$

In this case the parameters α enter the boundary conditions if, for example, current-voltage characteristics are to be determined (cf. [**3**]).

The solution manifold $G^{-1}(0)$ of (1.1) may have a very complex structure. In practical applications it is frequently desirable and sufficient to

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