ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 18, Number 1, Winter 1988

GENERALIZED HOMOGENEITY OF FINITE AND OF COUNTABLE TOPOLOGICAL SPACES

J.J. CHARATONIK AND W.J. CHARATONIK

ABSTRACT. Finite and countable topological spaces are investigated which are homogeneous, homogeneous with respect to open mappings or with respect to continuous ones. It is shown that for finite spaces all three concepts of homogeneity coincide, while for countable or for uncountable ones they are distinct. Some characterizations of countable spaces that are homogeneous in either sense are found for the metric setting.

0. Introduction. A topological space X with a topology (i.e., the family of open sets) $T(X)$ is said to be homogeneous with respect to a class M of mappings of X onto itself provided, for every two points $p, q \in X$, there is a mapping $f \in M$ such that $f(p) = q$. If M is the class of all homeomorphisms, we get the well-known concept of homogeneity of a topological space. A larger class of mappings than that of homeomorphisms (but not as large as the class of all continuous mappings) is one of open continuous mappings. Recall that a mapping $f : X \to Y$ between topological spaces is called open if images under f of open subsets of X are open in Y. And, finally, if M denotes the family of all continuous mappings of X onto X we get the concept of homogeneity with respect to continuity, that is due to David P. Bellamy.

Given a cardinal number k, let $D(k)$ and $I(k)$ denote a set of cardinality k equipped with the discrete and with the indiscrete topology, respectively. J. Ginsburg has proved in **[5]** that a finite topological space is homogeneous if and only if it is homeomorphic to the product $D(m) \times I(n)$ for some natural numbers m and n. The present paper has been inspired just by that short and nice result.

The paper is divided into two parts that concern finite and countable spaces, respectively (the term countable means of cardinality of the

Key words and phrases: continuous mapping, countable, finite, homogeneous, metric space, open mapping, regular space.
AMS *Subject Classification* (1980): Primary 54-01; 54C05; 54C10.
Received by the editors on December 16, 1985 and in revised form on June 17, 1986.