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TRANSLATION BY TORSION POINTS AND RATIONAL EQUIVALENCE OF O-CYCLES ON ABELIAN VARIETIES

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ABSTRACT. Let A be an abelian variety defined over an algebraically closed field. We give an elementary proof of the following result(Theorem 1): If γ is a 0-cycle of degree 0 on A, and $c \in A$ is a point of finite order, then γ is rationally equivalent to γ_c, the translate of γ under c. From this follows Theorem 2: Given any effective 0-cycle $\eta = (a_1) + \cdots + (a_r)$ on A, and any points of finite order $c_1, \ldots c_r \in A$ satisfying $c_1 + \cdots + c_r = o =$ the identity of A, we have that η is rationally equivalent to the 0-cycle $(a_1 + c_1) + \cdots + (a_r + c_r)$. Consequently, for $r \geq 2$, the set $E[\eta]$ of effective 0-cycles rationally equivalent to η is always at least a countably infinite set (Corollary 1). Further corollaries of Theorem 2 are given, including a generalization of Theorem 1 to higher dimensional cycles (Corollary 4).

0. Introduction. The purpose of this paper is to prove some elementary results concerning rational equivalence on abelian varieties, with an eye toward the problem of describing explicitly the set of all effective 0-cycles rationally equivalent to a given effective 0-cycle (see [**4**, pp. 133-135] for an indication of why this problem is interesting). We begin by establishing notation and then provide a summary of the contents and organization of the paper. Note that all varieties ($\Rightarrow$ irreducible) considered are defined over algebraically closed fields, and points are always closed points.

Let X be a nonsingular projective variety. If γ is a (pure) s-dimensional cycle on X, we write $[\gamma]$ for the set of all s-dimensional cycle γ' which are rationally equivalent to γ, written $\gamma \sim \gamma'$ (discussions of rational equivalence and related matters may be found in, e.g., [**1,3,4**]). The (Chow) group of s-dimensional cycles on X modulo rational equivalence is denoted $\mathrm{CH}_s(X)$. We write $E[\gamma] \subseteq [\gamma]$ for the set of all effective cycles $\eta \sim \gamma$.

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