ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 17, Number 4, Fall 1987

A NOTE ON TWO-GENERATOR GROUPS

MARTIN J. EVANS

Following J.L. Brenner and James Wiegold [**1**], let $\Gamma_1^{(2)}$ stand for the collection of all finite non-abelian groups G with the property that every non-trivial element is in a two-element generating set of G in which one element is of order two.

In [**1**] it is shown that $\mathrm{PSL}\,(2,q) \in \Gamma_1^{(2)}$ except when $q = 2, 3$ or 9. This led the above mentioned authors to ask whether almost all finite simple groups in $\Gamma_1^{(2)}$ are projective special linear groups.

In this note we answer this question negatively by showing that $\Gamma_1^{(2)}$ contains the Suzuki groups $Sz(2^{2n+1}), (n \geq 1)$. However, in the opposite direction we prove that the groups $\mathrm{PSL}\,(2,p^m)$, with p an odd prime, $p^m \neq 3$ or 9, are the only simple Chevalley groups over a field of odd characteristic that are contained in $\Gamma_1^{(2)}$.

Throughout the proof of the following theorem we use standard facts concerning Suzuki groups. These can be found in [**3**].

THEOREM 1. *Let $G = Sz(q)$ be a Suzuki group, where $q = 2^{2n+1}$ and $n \geq 1$. Then $G \in \Gamma_1^{(2)}$.*

PROOF. Given $x \in G$, we shall say that y is a *mate* for x in G if $\langle x, y\rangle = G$. Let Q be a Sylow 2-subgroup of G and let $z \in G$ be an involution not contained in Q. It follows from [**3**; Proposition 13] that each non-trivial element of odd order in G is conjugate to an element of the form πz where π is an involution in Q. In particular there exist involutions $\pi_1, \pi_2 \in Q$ such that $\pi_1 z$ is of order $q-1$ and $\pi_2 z$ is of order $q + r + 1$, where $r^2 = 2q$.

Let $x \in G$ be a non-trivial element of odd order. We aim to show that there exists an involution π_x such that $< x, \pi_x >= G$. Clearly it suffices to prove that some conjugate of x has a mate of order two in G. Therefore we may assume that $x = \pi z$ for some involution $\pi \in Q$. We distinguish two cases: (i) the order of x divides $q^2 + 1$; (ii) the order of

Received by the editors on April 24, 1986.