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BOUNDARY BEHAVIOR AND MONOTONICITY ESTIMATES FOR SOLUTIONS TO NONLINEAR DIFFUSION EQUATIONS

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ABSTRACT. The purpose of this article is to develop a number of estimates bearing on the boundary behavior of solutions to certain nonlinear diffusion equations. These estimates are then applied to show that the boundary behavior of solutions is related in a monotone way to the diffusion coefficient in the equation. This monotonicity may be applied in various ways to the analysis of inverse problems for nonlinear parabolic equations.

1. Compatibility of Overspecified Data Suppose $a(s)$ satisfies

$$\text{(1.1)}\qquad \begin{aligned} &\text{i) } a(s) \in C^1[0, \infty) \\ &\text{ii) } 0 < A_0 \leqq a(s) \leqq A_1 < \infty \text{ for } s \geqq 0 \\ &\text{iii) } 0 \leqq a'(s) \leqq A_2 \quad \text{for} \quad s \geqq 0 \end{aligned}$$

for a given set of constants, A_0, A_1, A_2. Then we may consider a nonlinear diffusion equation in which $a(s)$ plays the role of a coefficient.

$$\text{(1.2)}\qquad \partial_t u(x, t) = \partial_x(a(u)\, \partial_x u), \qquad 0 < x < 1, 0 < t < T.$$

Among the auxiliary conditions that $u(x, t)$ might be expected to satisfy as part of a well posed initial boundary value problem (IBVP) are the following

$$\text{(1.3)}\qquad \begin{aligned} &u(x, 0) = u_0, 0 < x < 1, \\ -&a(u)\, \partial_x u(0, t) = g_0(t), \quad u(0, t) = h_0(t), 0 < t < T, \\ &a(u)\, \partial_x u(1, t) = g_1(t), \quad u(1, t) = h_1(t), 0 < t < T. \end{aligned}$$

For u_0 a given non-negative constant, define

$$\text{(1.4)}\qquad \alpha(s) = \int_{u_0}^{s} a(\tau)\, d\tau, \qquad s \geqq u_0$$

for $a(t)$ satisfying (1.1). Then

$$\text{(1.5)}\qquad \alpha'(s) = a(s) \geqq A_0 \quad \text{for } s \geqq u_0$$

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