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SOME ANALOGUES OF A LEHMER PROBLEM ON THE TOTIENT FUNCTION

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1. Introduction and notation. In 1932, Lehmer [9] considered the equation

$$M\phi(n) = n - 1, \tag{1.1}$$

where $\phi(n)$ is the Euler totient function and asked whether the sets S_M of integers n satisfying (1.1) have any composite numbers. Obviously in the case $M = 1$, the answer is negative. But the problem is not settled for $M > 1$. However, the following partial solutions are known in the latter case. Firstly, Lehmer himself proved that each member of S_M is odd, squarefree and has at least seven distinct prime factors. Later Lieuwens [10], correcting the proof of Schuh [13], showed that $\omega(n) \geqq 11$ for every $n \in S_M$, where $\omega(n)$ denotes the number of distinct prime factors of n. Kishore [7] increased the lower bound of $\omega(n)$ to 13. Recently, Cohen and Hagis [2], using computational methods, established that $\omega(n) \geqq 14$. In another direction, Pomerance [12] proved that every such n is $< r^{2^r}$, where $r = \omega(n)$, and obtained that the number of $n \leqq x$ in any of S_M with $M > 1$ is

$$O(x^{1/2} \log^{3/4} x \cdot (\log \log x)^{-1/2}).$$

In this paper we discuss two analogous problems involving $J_k(n)$, the Jordan totient function of order k and $\phi^*(n)$, the unitary analogue of the Euler totient function. It is well-known that they are given by $J_k(1) = 1$, $\phi^*(1) = 1$, and if $n > 1$,

$$J_k(n) = n^k \prod_{p \mid n} \left(1 - \frac{1}{p^k}\right), \tag{1.2}$$

$$\phi^*(n) = \prod_{p^\alpha \| n} (p^\alpha - 1), \tag{1.3}$$

where the product in (1.2) is over prime divisors of n and that in (1.3) is

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