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FOURIER COEFFICIENTS OF NON-ANALYTIC AUTOMORPHIC FUNCTIONS OF SEVERAL VARIABLES

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1. Introduction. A well known theorem of H. Hamburger [**2**] states that the Riemann zeta function can be determined from its functional equation, under certain conditions of regularity. This result has been generalized by E. Hecke [**3**] to the zeta function of an imaginary quadratic field, over the field of rational numbers. Since then the general problem of determining all meromorphic functions, $\phi(s)$, which are expressible as a Dirichlet series absolutely convergent in some right half plane and satisfying functional equations of the type $\xi(s) = \xi(k - s)$, with

$$\xi(s) = \left(\frac{2\pi}{\lambda}\right)^{-s} (\Gamma(s))^a \left(\Gamma\left(\frac{s}{2}\right)\right)^b \left(\Gamma\left(\frac{s+1}{2}\right)\right)^c \phi(s),$$

has been studied. This problem has been solved for the functional equations of the type satisfied by the Dedekind zeta function for a real quadratic field over the field of rational numbers by H. Maass [**5**]. For this purpose Maass has introduced analogues of analytic automorphic functions, which he called non-analytic automorphic functions. Such functions are defined as complex valued functions, $f(\tau)$, of the two real variables x and y, with $\tau = x + iy$, satisfying the wave equation

$$\left[y^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \mu^2\right]f(\tau) = 0$$

in the upper half plane $y > 0$ and possessing transformation properties for the transformation group generated by the mappings $\tau \to \tau + \lambda$, $\tau \to -1/\tau$ (similar to analytic automorphic forms in the classical sense), and with the further requirement that $f(\tau)$ has growth restrictions as τ approaches the boundary of the upper half plane $\tau = x + iy$, $y > 0$. Later, Maass [**6**] generalized these functions of two real variables to functions of several variables and he called these functions non-analytic automorphic functions of several variables. The precise definition of these functions is as follows.

We consider the $k + 1$ dimensional hyperbolic space as a subspace of

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