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ON THE ESTIMATION OF EIGENVALUES OF HECKE OPERATORS

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Dedicated to the Memory of E.G. Straus and R.A. Smith

1. Introduction. Let $\mathbf{A}$ denote the adele ring of the rational numbers $\mathbf{Q}$ and suppose that we have a cuspidal automorphic representation π of $GL_2(\mathbf{A})$. (For the terminology and the details, we refer the reader to Gelbart [3] and Jacquet and Langlands [5]). Langlands [9] has described how one can attach an L-function to π. To describe this construction briefly, one can associate to π, a family of local representations π_p for each prime p of $\mathbf{Q}$. This family is uniquely determined by π such that

(i) π_p is irreducible for every p,

(ii) for all but finitely many primes p, π_p is unramified (that is, the restriction of π_p to $GL_2(\mathbf{Z}_p)$ contains the identity representation exactly once),

(iii) π can be factored as the restricted infinite tensor product $\pi = \bigotimes_p \pi_p$.

Let S denote the set of primes p for which π_p is ramified. For $p \notin S$, it is known that π_p corresponds canonically to a semisimple conjugacy class σ_p in $GL_2(\mathbf{C})$, where σ_p contains a matrix of the form

$$\begin{vmatrix} \alpha_p & 0 \\ 0 & \beta_p \end{vmatrix}.$$

If r denotes any finite dimensional complex representation of $GL_2(\mathbf{C})$, one can attach an L-series $L(s, \pi, r)$ as follows.

$$L(s, \pi, r) = \prod_p L(s, \pi_p, r),$$

where

$$L(s, \pi_p, r) = \det(1 - r(\sigma_p)p^{-s})^{-1}$$

whenever π_p is unramified. If π_p is ramified, and r is standard, we refer to J-L [5] for the definition of $L(s, \pi_p, r)$. It is known [9] that each $L(s, \pi, r)$

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