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ON MULTIDIMENSIONAL COVERING SYSTEMS OF CONGRUENCES

J. FABRYKOWSKI

Dedicated in memory of E. G. Straus

Let us consider a homogeneous system of congruences:

$$\sum_{j=1}^{k} a_{ij}x_j \equiv 0 \bmod m_i, \; 1 \leqq i \leqq n \tag{1}$$

where $m_i \geqq 2$ and

$$(a_{i1}, a_{i2}, \ldots, a_{ik}, m_i) = 1. \tag{2}$$

In [2] we have proved that if $n \geqq 2$ and a homogneneous system of the form (1) covers a k-dimensional cube $C_k \subset Z_k$ with the side length 2^{n-1} and such that $0 = [0, 0, \ldots, 0] \in C_k$ then it is a covering system, i.e., it covers every k-dimensional integer vector. We conjectured that the length $2^{n-2} + 2$ of the side of our cube is sufficient for the assertion and gave an example showing that the length $2^{n-2} + 1$ is not enough for the purpose.

In this paper we show that for a fixed number of variables and congruences we can check the conjecture by performing a finite number of operations.

In fact we shall prove the following:

THEOREM. *If there exists a homogeneous system of congruences of* $k \geqq 2$ *variables that covers a k-dimensional cube* C_k *with the side length* $2^{n-2} + 2$ *and such that* $0 \in C_k$ *which is not covering, then there exists a system (not necessary homogeneous) having the same properties which has all moduli less than* $2\max(k, 2^{n-2} + 2)(2^{n-2} + 2)^{k-1}$.

PROOF. Suppose that (1) covers the cube C_k, $0 \in C_k$ and is not covering. Certainly we can assume that no proper subset of our system has the same properties. We split indices $i \leqq n$ into three disjoint classes A, B, C as follows:

$i \in A$ if the i-th congruence is satisfied by $k + 1$ integer points from C_k which form a linearly independent set.

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