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PROBLEMS AND RESULTS ON CONSECUTIVE INTEGERS AND PRIME FACTORS OF BINOMIAL COEFFICIENTS

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To the memory of my old friend and coworker Ernst Straus

I have published many papers both alone and with coworkers during my long life on these and related questions. I give a partial list of these papers and will refer to them with roman numerals. In the last few years two important questions on consecutive integers were settled An old conjecture of Catalan stated that 8 and 9 are the only consecutive integers which are powers. Tijdeman [11] proved that there is a computable constant c so that above c there are no more consecutive powers.

Selfridge and I proved that the product of consecutive integers is never a power [I]. Probably both results can be strengthened. Selfridge and I conjectured that for $k \geqq 4$ there is a $p > k$ for which $p \parallel \prod_{i=1}^{k}(n+i)$ ($p \parallel m$ denotes $p \mid m$, $p^2 \nmid m$). Denote by $x_1 < x_2 < \ldots$ the sequence of powers. Presumably $x_{i+1} - x_i > x_i^{\varepsilon}$, but no proof of $x_{i+1} - x_i \to \infty$ has been published as yet.

Nevertheless very many simple unsolved problems remain. In this note I state some of my old unsolved problems which seem interesting to me and which were neglected and which do not seem to be completely hopeless. I also state some new problems and results and outlines of proofs.

Denote by $P(m)$ the largest and by $p(m)$ the least prime factor of m. Undoubtedly $P(m)$ and $P(m+1)$ are independent, but I can not even prove that the density of integers for which $P(m) > P(m+1)$ is $1/2$. (This problem seems very difficult and may be unattackable by our present methods.) Pomerance and I proved some preliminary results [3]. It is a simple exercise that the density of integers for which $p(m) > p(m+1)$ is $1/2$.

Let $n > k$ and put for $1 \leqq i \leqq k$

$$n + i = a_{n+i}(k) b_{n+i}(k),\ P(a_{n+i}(k)) \leqq k,\ p(b_{n+i}(k)) > k, \tag{1}$$

i.e., (1) gives the unique decomposition of $n + i$ as the product of two

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