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CHAIN CONDITIONS IN ENDOMORPHISM RINGS

ULRICH ALBRECHT

1. Introduction. Sometimes, a general theorem for a class of abelian groups can be proved by first showing that the property lives in the endomorphism rings. Then some theorems are proved about the groups using ring-theoretic properties of their endomorphism rings. This strategy was successfully used in [2], [3], and [4].

Whereas in [3] and [4], only torsion-free abelian groups of finite rank are considered, there is no bound on the ranks in [2]. However, this generalization requires the introduction of chain conditions on left ideals of the endomorphism ring $E(A)$ of the group A.

The central condition is one of these. It requires that every essential left ideal of $E(A)$ contains a central, regular element. If A has finite rank, this condition is equivalent to $E(A)$ being semi-prime. In general, if A satisfies the central condition, then $E(A)$ is a Goldie-ring, i.e., it has the ascending chain condition for left annihilators and finite left Goldie-dimension. These rings have been of interest in module-theory for the last few years since semi-prime Goldie-rings are exactly the rings with a semi-simple, Artinian left quotient ring.

In this paper, abelian groups A satisfying the central condition are considered from the point of view that they are a special class of Goldie-rings (Theorem 5.2). The requirement that $E(A)$ is a Goldie-ring is more natural than the central condition if A is not torsion-free. Therefore, this paper concentrates mostly on Goldie-groups, i.e., on abelian groups whose endomorphism ring is a left Goldie-ring.

Because the defining conditions of a Goldie-group behave quite differently with respect to decompositions in direct sums, they are studied in separate sections. In §2, it is shown that direct summands of an abelian group A such that $E(A)$ has the ascending chain condition for left annihilators have this property too (Proposition 2.1). Furthermore, an order-inverting, one-to-one correspondence between the left annihilators in $E(A)$ and certain subgroups of A is given (Theorem 2.5), as well as some applications of it.

In §3, abelian groups A are considered such that $E(A)$ has finite, left

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