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ON CONTINUED FRACTIONS CORRESPONDING TO ASYMPTOTIC SERIES

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ABSTRACT. Let $\{f_n\}$ be a sequence of complex-valued functions of a complex variable, each holomorphic at a point z_0 and meromorphic in a domain D containing z_0. Let $\{f_n\}$ correspond to a formal power series or to a formal Laurent series at z_0. [3, 148] Let a set $S \subset D$ and let z_0 be a limit point of S. Conditions are given for the functions f_n which insure that the corresponding series is the asymptotic expansion as $z \to z_0$, $z \in S$, of the limit of a subsequence of $\{f_n\}$. Applications are made to regular C-fractions, to general T-fractions, and to J-fractions.

DEFINITION. Let $\{f_n\}$ be a sequence of complex-valued functions of a complex variable, each holomorphic at a point z_0. Let $L = \sum_{k=0}^{\infty} c_k(z - z_0)^k$ be a formal power series, and let $G_m(z) = \sum_{k=0}^{m} c_k(z - z_0)^k$. The sequence $\{f_n\}$ is said to correspond to L at z_0, with order of correspondence ν_n, if there exists a sequence $\{\nu_n\}$ of positive integers such that $\nu_n \to \infty$ and

$$f_n(z) - G_{\nu_n-1}(z) = O((z - z_0)^{\nu_n}),$$

as $z \to z_0$.

DEFINITION. Let $\{f_n\}$ be a sequence of complex-valued functions of a complex variable, each holomorphic at ∞. Let $L = \sum_{k=0}^{\infty} c_k z^{-k}$ be a formal Laurent series, and let $G_m(z) = \sum_{k=0}^{m} c_k z^{-k}$. The sequence $\{f_n\}$ is said to correspond to L at ∞, with order of correspondence ν_n, if there exists a sequence $\{\nu_n\}$ of negative integers such that $\nu_n \to -\infty$ and

$$f_n(z) - G_{\nu_n+1}(z) = O(z^{\nu_n})$$

as $z \to \infty$.

A continued fraction with n^{th} approximant $f_n(z)$ is said to correspond to a formal power series or to a formal Laurent series if $\{f_n\}$ corresponds to the series.

THEOREM 1. *Let $\{f_n\}$ be a sequence of functions, holomorphic at z_0 and meromorphic in a domain D, with $z_0 \in D$. Let z_0 be a limit point of a set $S \subset D$. Let $\{f_n\}$ correspond to a formal power series $L = \sum_{k=0}^{\infty} c_k(z - z_0)^k$*

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