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THE SCHUR MULTIPLIER OF THE AUTOMORPHISM GROUP OF THE MATHIEU GROUP M_{22}

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In [1], Gagola and Garrison show that the Sylow 2-subgroups of the Schur multiplier of the automorphism group of M_{22} are cyclic of order two. In this note, we make use of this result to prove the following.

THEOREM. *The Schur multiplier of the automorphism group of the Mathieu group* M_{22} *is cyclic of order two.*

PROOF. Let α be a cocycle of $G = \text{Aut } M_{22}$ of odd prime order p. On restriction to M_{22}, α gives rise to a cocycle β say. Since $|G\colon M_{22}| = 2$, it follows by Satz IX of [5], that β also has order p. However, Mazat [4], has shown that the Schur multiplier of M_{22} is cyclic of order 12, so we deduce that p must be 3. Thus there is a group 3.G with a cyclic central subgroup A whose quotient is isomorphic to G and $3.G$ has a subgroup $3.M_{22}$ of index 2. Fixing a nontrivial irreducible character λ of A, the irreducible representations of $3.M_{22}$ which restrict to A as a multiple of λ may be regarded as projective representations of M_{22} and as such their characters are well-known (see [3]). The degrees of these irreducible representations are 21, 45, 45, 99, 105, 105, 210, 231, 231, 330 and 384. Since the irreducible Brauer characters modulo 3 of $3.M_{22}$ are precisely those of M_{22}, we may use the results of [3] to see that the restrictions of the characters 21 and 210 to 3-regular conjugacy classes give irreducible Brauer characters. We also note that the product character 21.21 has the decomposition

$$(1) \qquad 21.21 = 21 + 105_1 + 105_2 + 210$$

into irreducible characters.

The two classes of elements of order 11 in M_{22} fuse into one class in G. This means that the two characters of degree 105 (being exceptional for 11) give rise to an irreducible character of $3.G$ of degree 210. For each other irreducible representation D of $3.M_{22}$ whose restriction to A is a multiple of λ, there is a pair D^+, D^- of representations of $3.G$. If θ is the character of D, the characters θ^+ and θ^- agree on $3.M_{22}$ and $\theta^+(g) = -\theta^-(g)$ for all $g \in 3.G\backslash 3.M_{22}$. Thus (1) gives

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