ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 4, Fall 1984

SYSTEMS OF QUADRATIC FORMS II

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Dedicated to the memory of Gus Efroymson

0. Introduction. This paper is intended to continue the work of the papers [4] of the author and [3] of D. Leep. A system of r quadratic forms in n variables over a field F is replaced by a "quadratic map" $q: V \to W$ where V, W are F-vector spaces of dimension n resp. r. Section 1 contains some general definitions and properties of quadratic maps. In §2 I introduce the u-invariants $u_r (r = 1, 2, \ldots)$ and prove (or collect) results about these invariants. The main result of the present paper is given in §3. It concerns invariants u'_r which I hope are strongly related to u_r.

As the whole topic is still in "statu nascendi" it should be no surprise to the reader that there are more remarks, questions and problems than theorems. This paper owes a lot to discussions with D. Leep and D. Shapiro and in particular to a preprint of [3].

1. Generalities. Let F be an arbitrary (commutative) field with char $F \neq 2$, and let V, W be finite-dimensional F-vectorspaces.

DEFINITION 1. A map $q: V \to W$ is called quadratic if it has the following two properties:

a) $q(av) = a^2 q(v)$ for $a \in F$, $v \in V$

b) The map $b: V \times V \to W$ defined by

$$b(v_1, v_2) = q(v_1 + v_2) - q(v_1) - q(v_2)$$

is F-bilinear. It is called the "symmetric bilinear map associated to q".

DEFINITION 2. Two quadratic maps $q: V \to W$, $q': V' \to W'$ are called equivalent if there are F-isomorphisms $\sigma: V' \to V$, $\tau: W \to W'$ with $q'(v') = \tau(q(\sigma v'))$. This implies in particular that dim $V =$ dim V' and dim $W =$ dim W'.

REMARK. For dim $W = 1$ this reduces to similarity of the quadratic forms q, q' over F, not to ordinary equivalence of q, q'.

The radical Rad $q = \{v \in V \mid b(v, V) = 0\}$, regularity of q (i.e., Rad $q = 0$) and the (outer) direct sum of quadratic maps $q_i: V_i \to W$ are defined in an obvious way. For $q: V \to W$ the space $\underbrace{V \oplus \cdots \oplus V}_{m}$ is