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# INDUCING LATTICE MAPS BY SEMILINEAR ISOMORPHISMS

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In this paper all modules are left modules and all module homomorphisms act on the right. Ring homomorphisms are written on the left.

If $M$ is a module, let $\underline{\underline{L}}(M)$ denote the lattice of submodules of $M$. The Fundamental Theorem of Projective Geometry asserts that if $D$ and $K$ are two division rings and $\lambda: \underline{\underline{L}}(D^{(3)}) \to \underline{\underline{L}}(K^{(3)})$ is a lattice isomorphism between two three-dimensional free modules, then $\lambda$ is induced by a semilinear isomorphism. This means that there is an additive isomorphism $L: D^{(3)} \to K^{(3)}$ and a ring isomorphism $\sigma: D \to K$ such that $(X)L = \lambda(X)$ for each $X \in \underline{\underline{L}}(D^{(3)})$ and $(dV)L = \sigma(d)(V)L$ for all $V \in D^{(3)}$ and $d \in D$. For convenience the phrase "lattice isomorphism $\lambda: A \to B$" will be used to mean $\lambda: \underline{\underline{L}}(A) \to \underline{\underline{L}}(B)$ is a lattice isomorphism.

There has been some interest in generalizing this theorem to larger classes of rings. We prove here:

COROLLARY 6. *Let $n \geqq 3$. Let $R$ be any one of the following:*

1) *A serial ring (i.e., a finite product of rings, each of which has linearly ordered lattice of left ideals);*
2) *A semihereditary ring; or*
3) *An integral domain (not assumed to be commutative). Let $\lambda: R^{(n)} = \sum_{i=1}^{n} \oplus Ri_k \to S^{(n)}$ be a lattice isomorphism where the $\{i_k\}$ form a basis for $R^{(n)}$ and $Sa \approx S$, with $\lambda(Ri_k) = Sa$, for some $k$. Then $\lambda$ is induced by a semilinear isomorphism.*

We also show that if $R$ is an artinian ring of composition length $N$ and if $n \geqq N + 2$, then any lattice isomorphism $\lambda: R^{(n)} \to S^{(n)}$ which preserves cyclic submodules must be induced by a semilinear isomorphism. We actually need only that $\lambda$ preserves a small subset of the set of cyclic submodules of $R^{(n)}$. Note, since division rings have composition length 1, this generalizes the Fundamental Theorem. We also observe in the remarks before Lemma 1 that modulo lattice maps induced by certain kinds of projective modules, all such lattice maps preserve enough cyclic modules.

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