ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 1, Winter 1984

A METHOD FOR CONSTRUCTION OF SURFACES UNDER TENSION

GREGORY M. NIELSON AND RICHARD FRANKE

ABSTRACT. We describe and develop the properties of a method for interpolation of scattered data based on a generalization of the univariate spline under tension defined on a triangulation of the domain. Examples are given showing that the surface responds in a predictable way to the application of tension. As in the univariate case, tension parameters offer the promise of a way for the user to control the behavior of surfaces which have steep gradients implied by the data.

1. Introduction. We consider the problem of interpolation of scattered data, (x_i, y_i, z_i), $i = 1, 2, \ldots, N$. This requires the definition of a bivariate function F such that $F(x_i, y_i) = z_i$, $i = 1, 2, \ldots, N$. This problem has been addressed by several authors (see [5] and the references therein). To date, there are a number of methods available which work quite well on a wide variety of data sets, but these general purpose methods fail to yield acceptable results when steep gradients are implied by the data. A typical example is the data given in Table 1. The modified quadratic Shepard's method [6] is a good general purpose method. The result of applying it to the data of Table 1 is shown in Figure 1 and is similar to that obtained from other general purpose methods.

Our purpose here is to devise a scheme which allows some control over the behavior of the interpolant. In particular we wish to allow the capability to suppress, or at least to dampen the overshoot and undershoot of the surface near steep gradients. In the univariate case this has been achieved by various means, one being the spline under tension [20]. One type of bivariate analog of the univariate cubic spline is the minimum norm network of Nielson [15]. It is our goal to combine the two ideas in an attempt to obtain a certain type of bivariate analog of the spline under tension.

1980 *AMS subject classification*: Primary 65D05; Secondary 41A15.

Keywords and phrases: scattered data interpolation, spline under tension, multivariate approximation, surface fitting, data fitting, minimum norm.

Research of first author supported by ONR under contract # NR 044–433.

Copyright © 1984 Rocky Mountain Mathematics Consortium