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ON THE CHARACTER THEORY OF FULLY RAMIFIED SECTIONS

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1. Introduction. As corollaries of some lengthy and relatively difficult arguments one can derive several character theoretic facts which are quite often useful in the study of solvable groups. The purpose of this paper is to present a method which yields simple and direct proofs for some of these. As might be expected, we do not obtain the strongest possible results this way, but what we shall prove here is sufficient for many applications, some of which are discussed in the final section of the paper.

A configuration which often arises, and which will be our primary object of attention, is the following.

BASIC CONFIGURATION 1.1. *Let* $L \subseteqq K \lhd G$ *with* $L \lhd G$ *and* K/L *abelian. Let* $\varphi \in \mathrm{Irr}(L)$ *be invariant in* G *and assume* $\varphi^K = e\theta$ *for some* $\theta \in \mathrm{Irr}(K)$ *and integer* e.

Note that in this situation, $\theta_L = e\varphi$ and a computation of $\varphi^K(1)$ yields that $e^2 = |K: L|$. Also, θ is necessarily G-invariant. We occasionally discribe this situation by saying that K/L is a *fully ramified section in* G.

Frequently in these circumstances, we have in mind a particular subgroup H, such that $HK = G$ and $H \cap K = L$. (In other words, H is a complement for K in G relative to L.) We would like to obtain information which relates the irreducible characters of G which lie over φ with those of H which lie over φ. Generally, one must assume some additional hypotheses before being able to draw conclusions of the desired type.

The following is an example of a result of this kind. (We use the notation $\mathrm{Irr}(X|\varphi)$ to denote the set of irreducible constituents of φ^X where φ is an irreducible character of some subgroup of X.)

THEOREM 1.2. *Assume the Basic Configuration* (1.1) *and in addition suppose that at least one of* $|K: L|$ *or* $|G: K|$ *is odd. Then there exists* $U \subseteqq G$ *and a character* ψ *of* G *with* $K \subseteqq \ker \psi$ *such that*

a) $UK = G$ *and* $U \cap K = L$,

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