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# AN INVERSE PROBLEM FOR A PARABOLIC PARTIAL DIFFERENTIAL EQUATION

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**Introduction.** In this paper we shall study the problem of finding the coefficient $a(x)$ as well as the temperature $u(x, t)$ in the initial value problem

(0.1) $$u_t(x,t) - u_{xx}(x,t) + a(x)u(x,t) = 0, \; 0 < x < 1, \; 0 < t < T,$$

(0.2) $$u(0, t) = u(1, t) = 0, \quad 0 \leqq t \leqq T,$$

(0.3) $$u(x, 0) = f(x), \quad 0 \leqq x \leqq 1,$$

(0.4) $$u_x(0, t) = g(t), \quad 0 \leqq t \leqq T.$$

If the coefficient $a(x)$ were known, then (0.1)–(0.3) would constitute a well-posed problem for $u(x. t)$, but the indeterminate nature of the differential equation demands that we impose some additional boundary conditions and we have chosen to prescribe the flux, condition (0.4), at one end of the region.

Our methods will lead us to the classical inverse Sturm-Liouville problem, namely that of determining the potential $a(x)$ in the operator

(0.5) $$Ly = -y''(x) + a(x)y(x), \quad 0 \leqq x \leqq 1$$

where $y$ satisfies the boundary condition

(0.6) $$y(0) = y(1) = 0.$$

Typically in this problem one is given the spectrum $\{\lambda_n\}_{n=1}^{\infty}$ of $L$ (which in itself is insufficient to determine $a(x)$) plus some additional "Tauberian condition". This problem has received considerable attention and the Tauberian condition has taken a variety of forms. For example in [1], [5] a second complete spectrum $\{\mu_n\}_{n=1}^{\infty}$, arising from alternative self-adjoint boundary conditions, linearly independent to (0.6), was given. In [4], [5] it was assumed a priori that $a(x)$ is symmetric, that is $a(x) = a(1-x)$. In [3], the spectral function $\rho(\lambda)$ was specified, that is, that monotonic function with jump discontinuities at the points $\lambda = \lambda_1, \lambda_2, \ldots, \lambda_n, \ldots$ with the value of the jumps equal to $[\int_0^1 \phi_n^2(x)dx]^{-1}$ where $\phi_n(x)$

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