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CONVERGENCE IN FUZZY TOPOLOGY

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Abstract. Convergence and weak fuzzy continuity are developed and applied in fuzzy topological spaces.

Introduction. Lowen has skillfully used lower semicontinuous functions [5] and convergence [6] to obtain significant results about a proper subclass of the fuzzy topological spaces of Chang [1]. Lowen introduced this subclass in [5], also called its members fuzzy topological spaces and has adhered to the concept in his subsequent work. The main thrust of this paper is to take Lowen's ideas and results into work [2, 8, 9] based on Chang's paper. Often this requires methods which differ from Lowen's, since his work rests upon the usual topology of the unit interval, whereas Chang's viewpoint does not require a topology on the unit interval.

In §1 open fuzzy sets are described in terms of generalized lower semicontinuous functions which are used to characterize fuzzy continuous maps. Convergence is developed in §2 and then used to characterize fuzzy limit point and fuzzy continuity. An example is given to show that one of the characterizations of fuzzy continuity is the best possible. In §3 weak fuzzy continuity is given six characterizations which interestingly show its relation to other concepts, and the question of a complement for it is examined.

This paper assumes that the reader is familiar with the results in [6] and [9]. In general, the terminology and notation follow [9], except that fuzzy sets are denoted by lower case Greek letters.

1. F-continuity. If X is a set, then a fuzzy topology on X is a family T of mappings from X into [0, 1] such that the constant maps 0 and 1 are in T, the supremum of any subcollection of T is in T, and the infimum of any finite subcollection of T is in T. The members of T are called open fuzzy sets. A mapping between fuzzy topological spaces is called F-continuous if the inverse image of each open fuzzy set is open.

Several of the results in this section are based on the fact that if $f: X \to Y$ and $A \subset Y$, then the function $f^{-1}(\mu_A)$ maps X into $\{0, 1\}$ and

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