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OSCILLATION PROPERTIES OF FORCED THIRD ORDER DIFFERENTIAL EQUATIONS

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Introduction. A great deal of literature exists on the oscillation and nonoscillation of the equation

$$y''' + q(t)y = 0 \tag{1}$$

where $q(t)$ is a positive continuous function defined on $[0, \infty)$. However, little seems to be known about equations of the type

$$y''' + q(t)y = f(t) \tag{2}$$

where $f(t)$ is continuous and changes sign arbitrarily on $[0, \infty)$. The asymptotic properties of

$$y'' + q(t)y = f(t) \tag{3}$$

have been studied in several works, some which include the investigations of Burton and Grimmer [1], Keener [3] and Hammett [2]. Hammett, in particular, has given conditions under which the nonoscillatory solutions of (3) tend to zero. The main purpose of this work is to carry out a similar study for (2). The techniques used herein are patterned after those in [6] in which Singh concentrated on equations with retarded arguments.

Recall that a solution of (1) or (2) is called *oscillatory* if it has arbitrarily large zeros and nonoscillatory otherwise. A solution y is termed *quickly oscillatory* if there exists an increasing sequence of zeros of y, $\{t_i\}_{i=1}^{\infty}$ with the property that $\lim_{i\to\infty}(t_{i+1} - t_i) = 0$. The concept of quickly oscillatory solutions is also considered in other works, see [4] and [7].

Main result. It is well-known that if z is a nontrivial solution of $z'' + q(t)z = 0$ having at least two zeros on $[c, d]$, then $(d - c)\int_c^d q(t)dt > 4$. This inequality is sometimes called Lyapunov's inequality. Lovelady in [5] recently obtained analogous results for (1),

THEOREM 1. *If u is a nontrivial solution of (1) satisfying $u(a) = u(b) = 0$, and $u(x) \neq 0$ on (a, b), then*

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