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FREDHOLM ALTERNATIVES FOR NONLINEAR DIFFERENTIAL EQUATIONS

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Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

1. Introduction. During the past decade there has been much work devoted to existence theory for nonlinear boundary value problems which are of nonresonance type and many different types of problems and approaches to such problems have been discussed in the literature. In this paper we present an approach which unifies much which has been written about such problems. Our approach is based on some fixed point theorems which have their origin in the work of Lasota on nonlinear mappings which are not necessarily differentiable but have multivalued derivatives. In each of the applications we consider, we show that the problem at hand may be formulated in such a way that one of the fixed point theorems proved in §2 may be applied to deduce the existence of solutions. While much of the work is of a survey nature it turns out that many of the original proofs may be very much simplified and many of the results are established in a somewhat more general framework.

To illustrate the types of results discussed we present the following example.

Consider the nonlinear oscillator

$$x'' + g(x) = p(t), \tag{1.1}$$

where $p(t)$ is a 2π-periodic forcing term and g is a nonlinear restoring force such that

$$n^2 < \nu \leqq (g(x) - g(y))/(x - y) \leqq \mu < (n + 1)^2, x \neq y \tag{1.2}$$

where n is an integer. One sees that the unforced problem has, because of assumption (1.2), at most one solution, and as is to be seen, (1.1) has a 2π-periodic solution for any L^2 forcing term p. Thus one has a nonlinear Fredholm type alternative for such problems. As will be seen, assumption (1.2), allows us to formulate a fixed point problem

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