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ON CARATHEODORY CONDITIONS FOR FUNCTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAYS

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Dedicated to Professor Lloyd K. Jackson
on the occasion of his sixtieth birthday.

For functional differential equations of retarded type where the delay is fixed and finite, local existence results for initial value problems analogous to the Picard and Peano theorems for ordinary differential equations are well known; cf., for example, the book by J. Hale [3].

For initial value problems involving equations with infinite delays, the results of R. Driver [1] were perhaps the first to appear. More recently, existence theorems for such equations have appeared in papers by J. Hale and J. Kato [4], K. Schumacher [8], and F. Kappel and W. Schappacher [5]. In [1], [4], and [5] existence theorems of Peano-type, where solutions are continuously differentiable on their intervals of existence, are obtained for equations on quite general delay spaces. For such Peano-type existence theorems an important hypothesis that certain t-dependent composites of the function in the equation with translates of the state space functions be continuous seems to be crucial. For the state space CB consisting of continuous bounded functions on $(-\infty, 0)$ with supremum norm, it is known that such composites are not in general continuous, even for very smooth functions on CB; cf. [9] for an example. The example in [9], however, has a solution of Caratheodory type; i.e., a solution which is absolutely continuous on its interval of definition and satisfies the equation almost everywhere there.

Recently fairly general existence theorems for solutions of Caratheodory type have appeared; cf. [5], [8]. Earlier, A. Halany and J. Yorke [2] also stated such an existence theorem. As would be expected, a crucial condition in these results seems to be that the composites mentioned earlier be measurable.

In fact, recently other results involving Caratheodory type solutions such as continuous dependence of solutions on their initial functions also

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