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GENERALIZED PURE INJECTIVITY IN THE CONSTRUCTIBLE UNIVERSE

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In this note we establish for each infinite cardinal m the consistency of the statement "every m-pure injective abelian group is algebraically compact." More precisely, we show that this statement can be proved if we assume Gödel's Axiom of Constructibility. All of our results depend strongly on this axiom and, as is customary, we tag our theorems with the parenthetical notation ($V = L$) to indicate that they are derived in the $ZFC + V = L$ brand of set theory. It is actually Jensen's combinatorial principal $E(\kappa)$ which is our main tool, but we delay as long as possible introducing it.

Recall that a subgroup H of the abelian group G is said to be *m-pure* provided H is a direct summand of every intermediate subgroup K with $|K/H| < m$ and that an *m-pure injective* group is one which is a direct summand of every group containing it as an m-pure subgroup. Of course $\aleph_0$-purity is equivalent to ordinary purity (i.e., $nG \cap H = nH$ for all positive integers n) and the $\aleph_0$-pure injectives are just the well-known algebraically compact groups. In [4] we proved by an *ad hoc* argument that $\aleph_1$-pure injectives are algebraically compact, but were unable to obtain the same conclusion for any larger cardinals. In that paper we also found it convenient to consider certain weaker forms of generalized pure injectivity. In particular we called the group G an *(m, m)-pure injective* if it was a direct summand of any group K in which it appeared as an m-pure subgroup with $|K/G| = m$. That this concept is in fact genuinely weaker than m-pure injectivity was established in [4]. Indeed we exhibited there groups which were $(\aleph_1, \aleph_1)$-pure injective but not algebraically compact.

Throughout this note, m and κ denote infinite cardinals. By a *κ-filtration* of the group F we mean a family of subgroups $\{F_\alpha\}_{\alpha<\kappa}$ such that

(1) $\qquad |F_\alpha| < \kappa$ and $F_\alpha \subseteqq F_{\alpha+1}$ for all α,

(2) $\qquad F_\alpha = \bigcup_{\beta<\alpha} F_\beta$ if α is a limit ordinal, and

(3) $\qquad F = \bigcup_{\alpha<\kappa} F_\alpha$.

A subset of the cardinal κ is said to be *stationary* if it has nonempty inter-

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