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THE DUNFORD-PETTIS PROPERTY OF SOME SPACES OF AFFINE VECTOR-VALUED FUNCTIONS

PAULETTE SAAB

ABSTRACT. Let K be a Choquet simplex, E be a Banach space, let $C(K, E)$ denote the Banach space of all continuous E-valued functions defined on K with supnorm, and let $A(K, E)$ be the subspace of $C(K, E)$ consisting of affine functions. We show that $A(K, E)$ has the Dunford-Pettis property whenever $C(K, E)$ has the same property. We also exhibit a compact convex set C that is neither a Choquet simplex, nor a dual unit ball of a Banach space with the Dunford-Pettis property such that $A(C, \mathbf{R})$ has the Dunford-Pettis property.

Introduction. Let K be a compact convex subset of a locally convex Hausdorff space, and let E be a real or complex Banach space. In this paper we investigate the *Dunford-Pettis property* of $A(K, E)$, the space of all continuous and affine E-valued functions defined on a Choquet simplex K. This study is motivated by the fact that when K is a Choquet simplex, it is well known [**10**] that $A(K, \mathbf{R})^*$ is linearly isometric to an L^1-space, thus $A(K, \mathbf{R})$ has the Dunford-Pettis property. This raises the following interesting problem.

Problem. For a Choquet simplex K, and for a Banach space E, does $A(K, E)$ have the Dunford-Pettis property whenever E does?

It turns out that the above problem is closely related to another still open problem of whether the space $C(\Omega, E)$ of all continuous E-valued functions defined on a compact Hausdorff space Ω has the Dunford-Pettis property whenever E does. In this paper, we will show that, for a Choquet simplex K, the space $A(K, E)$ has the Dunford-Pettis property whenever $C(K, E)$ has the Dunford-Pettis property.

We also observe that there are compact convex sets Z such that $A(Z, \mathbf{R})$ does not have the Dunford-Pettis property. For it can be shown that a *real* Banach space V has the Dunford-Pettis property if and only if the space $A(B(E^*), \mathbf{R})$ of all continuous and affine functions on the unit ball

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