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MULTI-VALUED MAPPINGS OF CLOSED AND BOUNDED SUBSETS OF A NORMED LINEAR SPACE—A MAPPING DEGREE

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ABSTRACT. Previous extensions of degree theory to multi-valued mappings have required convexity or acyclicity conditions on the domains or point images of the mapping being considered. By using a straightforward combination of the results of D. G. Bourgin with results of K. Geba and A. Granas, a degree is defined in this paper which removes the acyclicity conditions, provided that the point images are acyclic in high enough dimensions. Using the degree, some fixed point theorems are developed.

0. Introduction. The study of non-acyclic upper semi-continuous multi-valued mappings was initiated by the work of D.G. Bourgin in [2]. Using a generalization of the Vietoris-Begle Theorem due to E. G. Skljarenko, [14], Bourgin defined the notion of a degree for a certain class of multi-valued mappings whose domain is the closed unit ball in a Banach space. In this paper, using a cohomology functor introduced by K. Geba and A. Granas, [8], the Bourgin degree is extended to include the class of compact non-acyclic upper semi-continuous multi-valued mappings with domains closed and bounded subsets of a normed linear space. Using this degree we obtain some fixed point theorems. Results similar to ours have been obtained using different methods by J. Bryszewski in the setting of Banach spaces where the closed and bounded subset is assumed to be either a closed ball or the boundary of a closed ball, [5].

We would like to recall that multi-valeud mappings have also been considered by: J. Bryszewski and L. Gorniewicz, [4], L. Gorniewicz, [10], A. Granas and J. W. Jaworowski, [11], R. Connelly, [6], for subsets of Euclidean space; F. E. Browder, [3], for subsets in Banach spaces; and L. Gorniewicz and A. Granas, [9], for acyclic multi-valued mappings.

1. Preliminaries. Let H^* denote the Čech cohomology functor with integer coefficients, $\mathbf{Z}$, from the category of metric spaces and continuous

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