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A NON-CATENARY, NORMAL, LOCAL DOMAIN

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Ever since Nagata constructed his celebrated example of a non-catenary Noetherian domain, it has been an open question whether or not such an example could be integrally closed. If the "chain conjecture" were valid, it could not be. However, in [3], T. Ogoma showed such an example existed. Precisely, he constructed a Noetherian domain R and showed that the integral closure of R was non-catenary and a finite R-module (thus Noetherian).

The present article is an alternate presentation of Ogoma's example. It is intended to serve two purposes. First, the construction itself has been simplified. It is shorter, requires less machinery, and should be more accessible than the original. Secondly, some new properties of R are observed. Most significantly, R is in fact integrally closed already (Theorem 4). This also simplifies matters.

It should be noted that [3] contains other examples, numerous positive results of interest, and a great deal of creativity which are omitted here. We begin the construction.

Let F be a countable field and $\{a_i, b_i, c_i \mid i \in \mathbf{Z}^+\}$ be indeterminates. Set $K_m = F(\{a_i, b_i, c_i \mid i \leqq m\})$ and $K = \bigcup K_m$. Let x, y, z, w be additional indeterminates.

Select a set $\mathscr{P}$ ($\subset K[x, y, z, w]$) of prime elements, exactly one for each height one prime of $S = K[x, y, z, w]_{(x,y,z,w)}$, such that $w \in \mathscr{P}$ and $\mathscr{P}$ contains infinitely many elements from $F[x, y, z, w]$.

Noting $\mathscr{P}$ is countable, these assumptions allow a numbering $\mathscr{P} = \{p_i \mid i \in \mathbf{Z}^+\}$ with $p_1 = w$ and $p_i \in K_{i-2}[x, y, z, w]$ for every $i \geqq 2$. Set $q_n = \prod_{k=1}^n p_k$, $f_n = x + \sum_{k=1}^n a_k q_k^k$, $g_n = y + \sum_{k=1}^n b_k q_k^k$, $h_n = z + \sum_{k=1}^n c_k q_k^k$, and $P_n = (f_n, g_n, h_n)S$ for $n \geqq 0$. Observe that for each $n > 0$, we have (modulo $((x, y, z, w)S)^2$) $f_n \equiv x + a_1 w$, $g_n \equiv y + b_1 w$, $h_n \equiv z + c_1 w$, and so f_n, g_n, h_n, w is a regular system of parameters. Therefore P_n is a height three prime ideal.

PROPOSITION 1. *$p_n \notin P_i$ where $i \geqq n - 1$.*

PROOF. Use induction on i. Since $w \notin (x, y, z)$, the proposition holds for $i = 0$ Assume in holds for $i - 1$. Thus $p_m \notin p_{i-1}$ for $m \leqq i$ and

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