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ON AUTOMORPHIC FORMS FOR THE GENERAL LINEAR GROUP

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ABSTRACT. Properties of Hecke operators for $GL(3, \mathbf{Z})$ are investigated as well as the analytic continuation of Eisenstein series for $GL(n, \mathbf{Z})$. The results described arose in the investigation of harmonic analysis of $GL(n, \mathbf{Z})$-invariant functions on the space of positive $n \times n$ real matrices.

1. Introduction. Automorphic forms for the general linear group, $GL(n, \mathbf{Z})$ of $n \times n$ integer matrices of determinant ± 1, can be viewed as relatives of the trigonometric functions—relatives which play a role in harmonic analysis on the Minkowski fundamental domain of positive definite $n \times n$ real matrices $\mathscr{P}_n$ modulo $GL(n, \mathbf{Z})$. Automorphic forms for $GL(n, \mathbf{Z})$ can also be considered to be related to Siegel modular forms which appear in the study of abelian integrals. The present paper is intended to be an expository discussion of some of the results needed to derive harmonic analysis on the fundamental domain $\mathscr{M}_n = \mathscr{P}_n/GL(n, \mathbf{Z})$ from that on $\mathscr{M}_{n-1}$. In order to remain at an expository level it will be considered legal to restrict to the case $n = 3$ (the case $n = 2$ being assumed known). For similar reasons the adelic interpretation will not be considered. §1 gives the properties of Hecke operators T_m for $GL(3)$ mostly. This includes the analytic continuation and Euler product of the L-function corresponding to an automorphic form for $GL(3, \mathbf{Z})$ which is an eigenfunction for all the Hecke operators. The analytic continuation of the L-function works for $GL(n)$, for all n (not just $n = 2, 3$), and is just a re-interpretation of work of Maass and Selberg. §2 studies the explicit analytic continuation of the L-functions and Eisenstein series for $GL(3)$ by a method which differs from that of Maass and Selberg in that it does without the differential operators introduced by Selberg. The method is close to one used by Arakawa, as well as to the adelic ideas of Jacquet and Shalika which appear in Inventiones Math. **38** (1976), 1–16. Helen Strassberg has also obtained such analytic continuations by adelic means. in **[20]**. §1 and §2 are closely related, since the Hecke operators for $GL(n)$ relate the two basic types of Eisenstein series for $GL(n)$. There is also a connection between

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