

OPEN COMPACT MAPPINGS, MOORE SPACES AND ORTHOCOMPACTNESS

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ABSTRACT. Two examples are given to show that an open compact map between zero dimensional Moore spaces need not preserve quasi-metrizability even if the domain space is separable or metacompact.

Perfect maps preserve quasi-metric spaces as well as non-Archimedean quasi-metric spaces and γ -spaces [14], [12]. The same is true for arbitrary closed maps with the first countable images [16], [13]. This paper is concerned with open and pseudo-open maps of quasi-metric spaces.

It was observed in [5] that quasi-metric spaces and γ -spaces are preserved under open finite-to-one maps; the corresponding result holds for non-Archimedean quasi-metric spaces. In answer to a question raised by R. F. Gittings [6], we show that a further generalization of these results is false; open compact maps do not preserve quasi-metrizability.

While the open compact images of metric spaces are the metacompact Moore spaces, which are very nice non-Archimedean quasimetric spaces [4], we show that one more application of an open compact map may yield a Moore space which is not quasimetrizable (Example 2). Hence there are non-quasi-metrizable spaces in MOBI, the smallest class containing all metric spaces and closed under open compact maps [1]. Example 2 answers a question asked by H. R. Bennett [2]. Example 1 shows that open compact maps do not preserve quasi-metrizability in the class of separable Moore spaces.

In both examples the domain spaces are non-Archimedean quasi-metric while the image is not quasi-metrizable and hence not γ , since developable γ -spaces are quasi-metrizable [8].

Since a developable space is orthocompact if and only if it is non-Archimedean quasi-metrizable [4], examples show that orthocompactness is not preserved under open compact maps. A Moore space may fail to be orthocompact even if it is an open compact image of a metacompact Moore

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