ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 11, Number 3, Summer 1981

MEROMORPHIC STARLIKE FUNCTIONS

PAUL J. EENIGENBURG† and ALBERT E. LIVINGSTON

ABSTRACT. Let $\Lambda^*(p)$ the class of functions $f(z)$ univalent and meromorphic in $\Delta = \{z|\ |z| < 1\}$ with simple pole at $z = p$, $0 < p < 1$, $f(0) = 1$ and which map Δ onto a domain whose complement is starlike with respect to the origin. We discuss the coefficients of the Taylor series $f(z) = 1 + \sum_{n=1}^{\infty} a_n z^n$, $|z| < p$ and the Laurent series $f(z) = \sum_{n=-\infty}^{\infty} b_n z^n$, $p < |z| < 1$. We also obtain best possible order estimates on $L(r)$, the length of the image of $\{z:|z| = r\}$ for a function in $\Lambda^*(p)$. Estimates on the integral means of higher order derivatives are also obtained and in the last section a question of Holland [5] is answered.

1. **Introduction.** Let $\Sigma(p)$ denote the class of functions $f(z)$ which are meromorphic and univalent in $\Delta = \{z||z| < 1\}$ with a simple pole at $z = p$, $0 < p < 1$, and with $f(0) = 1$. If, further, there exists δ, $p < \delta < 1$, such that

$$\operatorname{Re} \frac{zf'(z)}{f(z)} < 0 \tag{1.1}$$

and

$$\frac{1}{2\pi} \int_0^{2\pi} \operatorname{Re} \frac{zf'(z)}{f(z)}\, d\theta = -1 \tag{1.2}$$

for $\delta < |z| < 1$ with $z = re^{i\theta}$, we say that $f(z)$ is in $\Lambda(p)$. Functions in $\Lambda(p)$, which have been discussed in [**10**, **11**], map Δ onto a domain whose complement is starlike with respect to the origin. However, there exist functions with pole at p having this mapping property which do not satisfy (1.1) if $p > 1/2$. The function

$$F(z) = \frac{-p(1+z)^2}{(z-p)(1-pz)}$$

maps Δ onto the complement of the interval $[-4p/(1-p)^2, 0]$ but does not satisfy (1.1) if $p > 1/2$ [**10**].

Let $\Lambda^*(p)$ denote the class of functions $f(z)$ which have the representation

†The research of the first author was supported by a Western Michigan University Research Fellowship.

Received by the editors on June 21, 1979, and in revised form on January 11, 1980.