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COUNTABLE UNIONS OF 0-DIMENSIONAL DECOMPOSITIONS

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In this paper we shall investigate monotone upper semicontinuous decompositions of a locally compact connected metric space, where the projection of the nondegenerate elements is 0-dimensional.

In particular we show that if G can be described by a countable collection G_i of decompositions of M which fit together in a "semicontinuous" manner and where each G_i is shrinkable, then G is shrinkable and M/G is homeomorphic to M.

Throughout this paper M will denote a locally compact connected metric space, G a monotone 0-dimensional upper semicontinuous decomposition of M, P the projection map of M onto the decomposition space M/G, and $H(G)$ the set of all nondegenerate elements of G. By 0-dimensional we mean that $P(H(G))$ is a 0-dimensional subset of M/G. The closure of a set X will be denoted by $\mathrm{Cl}(X)$, its boundary by $\mathrm{Bd}(X)$, and the set of all points within ε of X by $N(X, \varepsilon)$.

A 0-dimensional decomposition G is shrinkable if for each open set B containing the union of the nondegenerate elements, each positive number ε, and each homeomorphism f from M onto M, there is a homeomorphism h from M onto M such that the diameter of $h(f(g))$ is less than ε for each g in G and $f(x) = h(f(x))$ for x an element of $M - f(B)$. This definition of shrinking was shown to be equivalent to the definition which uses the identity map for f (see [1] and [3]).

Since it will be used repeatedly we restate Theorem 1 of [3].

THEOREM 1. *Let K be an open covering of $H(G)$ in M, then there exists a refinement K' of K such that*

1) *K' is an open covering of $H(G)$ in M,*
2) *K' is a disjoint countable collection,*
3) *If X is a compact set, then the closure of $\{k \in K' | k$ intersects $X\}$ is a compact set, and*
4) *K' is a locally null collection.*

THEOREM 2. *If G is a monotone upper semicontinuous decomposition of a locally compact connected metric space M such that $P(H(G))$ is a 0-dimensional subset of M/G and there exists a countable collection $\{G_i\}$ of shrinkable upper semicontinuous decompositions of M such that*

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