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A SINGULAR PERTURBATION APPROACH TO NONLINEAR SHELL THEORY

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Introduction. The purpose of the present paper is twofold: On the one hand we want to investigate a problem naturally arising in the nonlinear theory of shallow shells; and on the other hand we hope that the method we are going to employ may prove useful in other applications, where a constructive existence proof for differential equations with small parameters is to be given by asymptotic expansions.

We shall deal with the basic system of nonlinear fourth order partial differential equations describing the behaviour of a gently sloping shell subject to an external load. These are known as Marguerre's equations, a derivation of which is given by Weinitschke in [**19**]. We shall utilize them in nondimensionalized form, given in §1, thus introducing a parameter ε which characterizes the thickness of the shell and multiplies the highest order derivatives (see [**15**]).

Now let the shell with its edge simply supported be exposed to a sufficiently strong vertical pressure. The question is whether the shell returns to its initial state, or remains in a deflected position after the load is removed. To put it mathematically: Do Marguerre's equations possess stable nontrivial solutions besides the trivial one for vanishing external load?

This problem was investigated by Srubshchik in [**15**]. The above mentioned feature suggests, for small ε, viewing it as a singular perturbation problem which is solved formally by an asymptotic method invented in the pioneering work of Višik and Lyusternik [**18**]. Since then, this method has been successfully used by several authors dealing with second order equations, more recently even in the nonlinear case (cf. Fife [**5**]).

The asymptotic expansions for a nontrivial solution constructed in [**15**] of course satisfy the differential equations approximately, the same being true for the boundary conditions. Yet they do not satisfy the boundary conditions corresponding to a simple support of the shell exactly, a defect which is typical for those conditions comprising derivatives of different order. This is the reason the justification of the formal approximations given in [**15**], i.e., the proof of the existence of an actual solution in their

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