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QUOTIENTS OF $C(X)$ BY UNIFORM ALGEBRAS

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ABSTRACT. If A is a uniform algebra on X and M is a closed A-submodule of $C(X)$, the following are shown: that $C(X)/A$ is not separable provided A is antisymmetric and X is totally disconnected and non-metrizable (in particular giving yet another proof that L^∞/H^∞ is not separable); that $C(X)/A$ is not reflexive unless $A = C(X)$; and that, at least under a suitable additional hypothesis on A, $C(X)/M$ is not reflexive if M has infinitecodimension in $C(X)$.

1. **Introduction**. Several years ago, E. Berkson and L.A. Rubel published an article entitled "Seven different proofs that L^∞/H^∞ is not separable" [1]. The present paper consists of an eighth proof (which appears to be the simplest available) from the point of view of function algebras, and of a number of results arising from investigations related to or suggested by it. Ultimately, the objective is to study the Banach space structure (there is no obvious algebra structure) of $C(X)/A$, where A is a uniform algebra on X. We shall show that under certain conditions $C(X)/A$ cannot be separable, and that (if $A \neq C(X)$) it cannot be reflexive. We shall also briefly consider $C(X)/M$ for M a closed A-submodule of $C(X)$.

2. **Non-separability**. By a uniform algebra A on the compact Hausdorff space X we mean a (uniformly) closed subalgebra of $C(X)$ such that A separates the points of X and contains the constant functions (the latter is essentially irrelevent for most of what follows). As is well known, L^∞ can be viewed as $C(X)$ for a certain totally disconnected non-metrizable compact Hausdorff space X, and H^∞ as a uniform algebra A on this X; the last chapter of K. Hoffman's book [4] serves as a good reference on this representation. It is also well known that this A is antisymmetric, that is, the only real-valued functions in A are the constants. Thus the non-separability of L^∞/H^∞ is a particular instance of the following theorem.

THEOREM. *Let A be an antisymmetric uniform algebra on a compact Hausdorff space X which contains uncountably many distinct open-and-closed sets (which happens, for example, if X is totally disconnected and non-metrizable, and in any case implies non-metrizability of X). Then $C(X)/A$ is not separable.*

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