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A CONSTRUCTIVE APPROACH TO KERGIN INTERPOLATION IN R^k: MULTIVARIATE B-SPLINES AND LAGRANGE INTERPOLATION

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ABSTRACT. In this paper we construct an interpolation map from $C^{(n)}(R^k)$ into $\pi_n(R^k)$ (polynomials of total degree $\leqq n$) whose existence and uniqueness was proved by P. Kergin. We also provide numerically efficient and stable recurrence relations for computing the multivariate B-spline and its derivatives.

0. **Introduction.** This paper naturally divides into two distinct but intertwining parts. The results presented here find their origin in a recent result of P. Kergin.

In his doctoral thesis P. Kergin [3] introduced a novel method of interpolating smooth functions of several variables. This method is a natural multivariate version of Lagrange interpolation. In §1 of this paper we give an explicit representation for Kergin's interpolation scheme. (P. Milman independently derived this formula, see our remarks in the acknowledgements.) This should be contrasted with Kergin's *existence* theorem, stated below as Theorem 1.

Divided differences and Lagrange interpolation are closely related. Through B-splines, a divided difference has an integral representation on smooth functions. §2 of this paper draws upon another idea whose source is a result of Curry and Schoenberg. They gave a geometric interpretation of B-splines which led to an interesting multivariate version of this function [2]. In that paper, de Boor raises the question of providing recurrence relations for computing these multivariate functions. In the second part of the paper we provide such recurrence relations.

1. **Multivariate Lagrange Interpolation.** We let $\pi_n(R^k)$ denote the space of polynomials of total degree $\leqq n$ and $C^{(n)}(R^k)$ be all functions with n continuous derivatives on R^k.

Kergin proved the following theorem.

THEOREM 1. *Given any* $x^0, \ldots, x^n \in R^k$, *not necessarily distinct, there is a*

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