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APPROXIMATING MAPS INTO FIBER BUNDLES BY HOMEOMORPHISMS

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1. **Introduction.** By a *manifold* we will mean either a finite-dimensional topological n-manifold or a Q-manifold, i.e., a manifold modeled on the Hilbert cube Q. Let $p : E \to B$ be a fiber bundle and let $f : M \to E$ be a map, where M, E and B are all manifolds. In this paper we will be interested in the following general question: *When is f homotopic to a homeomorphism $h : M \to E$ so that ph is close to pf?* Our main results in this direction are Theorem 1, which concerns Q-manifolds, and Theorem 3, which concerns n-manifolds. In Theorem 2 we apply Theorem 1 to the problem of approximating approximate fibrations of Q-manifolds by fiber bundle projections. Theorems 4 and 5 are applications of Theorem 3. Theorem 4 is a new proof of Goad's result on approximate fibrations of n-manifolds [10] and Theorem 5 is a new proof of the codimension 2 tubular neighborhood theorem of Kirby-Siebenmann for n-manifolds [13].

In order to state our results we will need some definitions. All spaces in this paper will be locally compact, separable and metric, unless otherwise stated, and a *proper* map is a map for which preimages of compacta are compact. If α is an open cover of a space Y, then a proper map $f : X \to Y$ is an *α-equivalence* if there is a map $g : Y \to X$ so that (1) fg is α-homotopic to the identity and (2) gf is $f^{-1}(\alpha)$-homotopic to the identity. Statement (1) means that the track of each point of Y under the homotopy $fg \simeq id$ lies in some element of α, and (2) means that the track of each point of X under the homotopy $gf \simeq id$ lies in some element of $f^{-1}(\alpha) = \{f^{-1}(U) \mid U \in \alpha\}$. It follows from [11] that if X and Y are ANRs and f is a CE map (i.e., f is proper, onto, and preimages of points have trivial shape), then f is an α-equivalence, for every α. Here is our main result for Q-manifolds.

THEOREM 1. *For each open cover α of a Q-manifold B there is an open cover β of B so that if $p : E \to B$ is a fiber bundle, with fiber a compact* ANR *for which π_1 of each component is free abelian, then any $p^{-1}(\beta)$-equivalence from a Q-manifold M to E is $p^{-1}(\alpha)$-homotopic to a homeomorphism.*

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