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A REMARK ON UNIFORM CLASSIFICATION OF BOUNDEDLY COMPACT LINEAR TOPOLOGICAL SPACES

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ABSTRACT. Only linear spaces over reals are considered. It is proved that (i) if a locally convex space is uniformly homeomorphic to a Montel-Fréchet space then it is isomorphic to it and (ii) if two separable conjugate real Banach spaces are uniformly homeomorphic with respect to their w^*-topologies then they are isomorphic.

Introduction. It seems that we are quite far from understanding under what circumstances the existence of a uniform homeomorphism between linear topological spaces over the reals implies that the spaces are isomorphic. Very recently, Aharoni and Lindenstraus [1] have shown that in general this is not the case. they Namely, exhibited an example of two nonisomorphic but Lipschitz homeomorphic nonseparable, nonreflexive Banach spaces. On the other hand, it was known, for example, that if one of the spaces involved is either a Hilbert space [7] or the Fréchet space of all real valued sequences [8] then the existence of a uniform homeomorphism implies that the spaces are isomorphic. Also, recently Ribe [10] has proved that if two Banach spaces are uniformly homeomorphic then they have the same local structure.

The aim of our note is to prove that (i) if a locally convex linear topological space is uniformly homeomorphic to a Montel-Fréchet space then it is isomorphic to it, and (ii) if two separable conjugate Banach spaces are uniformly homeomorphic with respect to their w^*-topologies then they are isomorphic.

We shall consider vector spaces over the field of reals only. In what follows E, F will stand for locally convex topological vector spaces with $\mathscr{P}$ and $\mathscr{Q}$ the corresponding systems of all continuous pseudonorms. Spaces E, F are supposed to be uniform spaces with their natural translation-invariant uniformity. We shall call the mapping $T: E \to F$ Lipschitz if for each $q \in \mathscr{Q}$ there is $p \in \mathscr{P}$ such that $q(Tx - Ty) \leqq p(x - y)$ for all x, y in E.

We shall often use the following lemma, stating that a uniformly continuous map between localy convex spaces has the Lipschitz property for large distances. It is a simple special case of a result of Corson and Klee [5].

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