ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 10, Number 1, Winter 1980

THE DIVERGENT BEAM X-RAY TRANSFORM

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1. **Introduction.** Until recently there was little need for mathematics in radiology. Films were examined individually, and by eye, and mathematics had little to offer to the procedure. The picture changed radically in the late 1960's with a breakthrough in radiology called Computed Tomography [**10**] in which the attenuation in the x-ray beam is measured in an extremely sensitive quantitative way, and the information from many x-rays from different sources is assembled and analysed on a computer. In this new situation mathematics can make significant practical contributions concerning the nature of the total information conveyed by x-rays from many sources, the extent to which this information determines the object x-rayed, suitable methods for using the information to build a detailed reconstruction of the object, etc.

Mathematically, the divergent beam x-ray transform, or radiograph, of a function f on R^n from a source point a is the function $\mathscr{D}_a f$ defined by

$$(1.1) \qquad \mathscr{D}_a f(\theta) = \int_0^\infty f(a + t\theta)\, dt \text{ for } \theta \in S^{n-1}.$$

Physically, f is the density function of the object x-rayed, so that $\mathscr{D}_a f(\theta)$ is the total mass of the object along the half line with origin a and direction θ. In practice this number is determined by measuring the attenuation in the x-ray beam along the half line. The basic problem from either the practical or the mathematical point of view, is the extraction of information about the unknown function f from knowledge about certain of the radiographs $\mathscr{D}_a f$, $a \in A$. Throughout the article it is assumed that f is integrable and that f vanishes outside a bounded open set Ω with closure $\bar{\Omega}$ and closed convex hull $\hat{\Omega}$.

Received by the editors on April 13, 1978.