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ASYMPTOTIC EXPANSIONS IN PERFORATED MEDIA WITH A PERIODIC STRUCTURE

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Dedicated to N. ARONSZAJN

0. Introduction. We consider elliptic equations

$$Au_\varepsilon = f \tag{1}$$

in domains Ω_ε which consist of a perforated medium, with a "large" number of holes or of obstacles of "size" ε and which are arranged in a periodic manner, also with period ε. In (1) u_ε is subject to some boundary conditions, and we want to study the behaviour of u_ε as $\varepsilon \to 0$.

This problem has already been considered by L. Tartar [**14**], D. Cioranescu [**5**], and D. Ciroranescu and J. Saint Jean Paullin [**6**] by energy methods; one obtains in this manner the behaviour of u_ε as $\varepsilon \to 0$, and the periodic structure is *not* used in an essential manner. For situations where the "volume" occupied by the holes is "smaller" than in the present case, cf. V. A. Marcenko and E. Yu. Hruslov [**12**] and Rauch and Taylor [**13**].

In this paper we show that—by using this time the periodic structure in an (apparently) essential manner—one can obtain much more, that is, under suitable hyopthesis on f, one can obtain an expansion of any order in ε. We will construct functions $u_0, u_1, u_2, \ldots$ such that

$$u_\varepsilon - (u_0 + \varepsilon u_1 + \cdots \varepsilon^m u_m)$$

is of order ε^m *in a Sobolev space on* Ω_ε. Actually, in the situations considered here $u_0 = 0$, $u_1 = 0$.

The method used here is a variant of the method of multi-scales as used in the book A. Bensoussan, J. L. Lions and G. Papanicolaou [**4**] (and as anticipated by J. Keller) for problems of homogenization arising in composite materials (We refer to the book just quoted for bibliographical references, in particular to the work of de Giorgi, Spagnolo and their associates, Bakhbalov, Babuska, Murat and Tartar.) The new part here is that in some case, boundary layer terms can be avoided. (The construction of boundary layer terms, when they are needed, is a largely open question in Composite Materials as well as in Perforated Media.) The structure of the expansion in perforated media has been briefly given in the lecture [**9**] of the Author in Poland and in lectures in the Collège de France, Fall 1977.

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