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INFINITE GROUPS WITH A SUBNORMALITY CONDITION ON INFINITE SUBGROUPS

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1. **Introduction and notation.** If $\mathfrak{X}$ is a subgroup theoretic property, let $(\mathfrak{X})$ denote the class of groups all of whose subgroups are $\mathfrak{X}$-subgroups, and let $I(\mathfrak{X})$ denote the class of all infinite groups, all of whose infinite subgroups are $\mathfrak{X}$-subgroups. In [4] and [5] Černikov studies the structure of groups in three classes (we do not require a trivial group in a class) of the form $I(\mathfrak{X}) - (\mathfrak{X})$, for $\mathfrak{X}$ denoting normal, ascendant, and complemented, respectively. In [14], R. Phillips studies a class of this form for $\mathfrak{X}$ denoting serial; this class is the same as for $\mathfrak{X}$ denoting ascendant. In the present paper we study the structure of $I\mathfrak{P} - \mathfrak{P}$, where $\mathfrak{P}$ is the class of all groups, all of whose subgroups are subnormal of bounded defects, and where $I\mathfrak{P}$ is the class of all infinite groups, all of whose infinite subgroups are subnormal of bounded defects.

Our major result is that locally nilpotent groups in $I\mathfrak{P} - \mathfrak{P}$ are Černikov groups and we obtain a structure theorem for them in § 2. By studying certain automorphisms of divisible abelian p-groups of finite rank in § 3, we further characterize locally nilpotent groups in $I\mathfrak{P} - \mathfrak{P}$ in terms of direct limits of p-groups of maximal class in § 4. We explain why we restrict our attention to locally nilpotent groups following Theorem 2.4.

$\mathfrak{N}$, $\mathfrak{N}_c$, $L\mathfrak{N}$, Min, Z, ZA, and ZD denote the classes of nilpotent groups, nilpotent groups of class at most c, locally nilpotent groups, groups satisfying the minimal condition or descending chain condition on subgroups, groups having a central series, hypercentral groups, and groups with a descending central series, respectively. A group of type p^∞ is a group with generators $x_1, x_2, \cdots$ and defining relations $px_1 = 0$, $px_{n+1} = x_n$.

A divisible abelian group of finite rank is a direct sum of finitely many groups each of which is the full rational group or a group of type p^∞ for various primes p; if such a group is a p-group the rank is the number of summands. A Černikov (or extremal) group is a finite extension of an abelian group in Min. A Černikov group G possesses a characteristic divisible abelian group $D(G)$ of finite rank and finite index. $H \leqq G$ and $H < G$ denote that H is a subgroup of G and H is a proper subgroup of G, respectively. $H \operatorname{sn} G$, $H \operatorname{sn}_r G$, $H \operatorname{ser} G$, and

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