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A METHOD FOR THE APPROXIMATE SOLUTION OF SOME STOCHASTIC EQUATIONS OF POPULATION BIOLOGY

BENJAMIN S. WHITE

ABSTRACT. A method is presented for analyzing several types of equations describing interacting species in a random environment. Approximate solutions of the models are obtained under the assumption that the stochastic fluctuations of the environment are rapidly varying.

1. This paper contains a brief description of a method which has been used to analyze several types of stochastic models of interacting species. Theorems justifying the method are in [**1**, **4**, **6**], and a more detailed discussion of the applications is in [**5**, **2**].

Among the simplest deterministic models of interacting species is the ordinary differential equation. Let $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))$ be a vector of (scaled) population sizes $x_1, x_2, \cdots, x_n$ of a system of n species. Then in a non-random environment, the equations are

$$\frac{d}{dt} x(t) = F(x(t), \gamma),\ x(0) = x_0 \text{ given.} \tag{1.1}$$

Here F is some non-linear function describing the species interaction dynamics, and $\gamma = (\gamma_1, \cdots, \gamma_m)$ is a vector of m parameters occurring in the equations, e.g., growth rates, carrying capacities, inter-species competition coefficients, etc. More realistic models than (1.1) may also incorporate time-delays, as well as an environment which is itself changing in time. Further generalizations of (1.1) may treat spatial inhomogeneities in the environment.

In a stochastic environment, the parameters γ may be expected to fluctuate randomly in time. The fundamental assumption that we make here is that the random environmental fluctuations are on a time scale which is much faster than any other time scale in the sytem. Mathematically, we assume that $\gamma = \gamma(\tau)$ is a stochastic process, and that

$$\tau = t/\epsilon,\ 0 < \epsilon \ll 1. \tag{1.2}$$

The small parameter ϵ is then the ratio of the time scale t for which the equations are written to the time scale τ of the noisy fluctuations. The idea is to exploit this disparity of time scales to derive an approx-

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