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ENVELOPING W^*-ALGEBRAS

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ABSTRACT. Starting witth a C (complex)-algebra R having a conjugate linear (idempotent) involution on R, and a set P of positive C-linear functionals on R (with no topology on R), the C^* and W^*-enveloping algebras of R are constructed. They are uniquely determined by universal mapping properties (Theorem IV).

Let S be an involutive semi-topological semigroup (not necessarily locally compact), H any Hilbert space, and $L(H)$ the W^*-algebra of all bounded operators in the σ (ultraweak) topology. As a special case with $R = \mathbf{C}S$ the semigroup algebra, and with P arising from a set of bounded continuous positive definite functions on S, the C^* and W^*-enveloping algebras $\mathbf{A}S$ and $\mathbf{W}S$ of S are obtained. There is a map $S \rightarrow S/\Omega$, and $S \rightarrow S/\Omega \subset \mathbf{A}S \subset \mathbf{W}S$, where S/Ω is the image of S in $\mathbf{A}S$. Then $\mathbf{W}S$ is uniquely determined by the universal property that any σ-continuous representation $S \rightarrow L(H)$ factors uniquely through a (σ-continuous homomorphism $S \rightarrow \mathbf{W}S \rightarrow L(H)$ of the W^*-algebras $\mathbf{W}S$ and $L(H)$.

0. **Introduction.** There is such a large number of duality theories for various classes of semigroups and groups, some of them overlapping, that old ones are as quickly forgotten as new ones are invented. Thus any new development that would in some way tend to simplify, unify, and generalize these would be welcome. A duality can perhaps be regarded as a functor from some category of semigroups to some well defined category of C^* or W^*-algebras. For example, in [8] this functor is an equivalence of categories.

Such a functor should preserve as much of the available structure on the given category of semigroups or groups as possible. Although the direct product of semigroups in itself is of minor interest, its importance lies in the fact that in subsequent generalizations of duality it should single out the appropriate subcategory for the equivalence among all the W^*-algebras, namely those that carry an additional coalgebra structure. Thus the direct product of semigroups should map into a tensor product of W^*-algebras.

Frequently, duality theories ([6] and [9]) use C^* and W^*-tensor products that do not have the usual universal mapping property. Here we will be forced to use the categorical W^*-tensor product ([7]); others simply would not work because they lack the above multiplicative property. Also, here the W^*-tensor product will have to be defined in terms of a (tensor) product on the preduals of W^*-algebras.

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