

NEIGHBORHOODS, BASES AND CONTINUITY IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT. It is shown that fuzzy continuous functions can be characterized by the closure of fuzzy sets, a subbasis for a fuzzy topology and fuzzy neighborhoods. Additional results are obtained concerning the collection of all fuzzy topologies on a fixed set, the interior of a fuzzy set, the closure of a fuzzy set, a fuzzy limit point, the derived fuzzy set and the relative fuzzy topology.

1. **Introduction.** Fuzzy topological spaces were first introduced in the literature by Chang [1] who studied a number of the basic concepts, including fuzzy continuous maps and compactness. Nazaroff [4] has used the fuzzy topological machinery of Chang as the starting point for developing a generalized theory of optimal control and has contributed the basic ideas of exterior and closure of a fuzzy set. Within a broader framework, Goguen [2] presented the fundamental ideas of basis, subbasis and product in an investigation of compactness. Fuzzy spaces are surveyed by Wong [6].

This paper is a thorough study of the basic notions in fuzzy topological spaces. We establish six characterizations of fuzzy continuous maps by introducing the notion of a neighborhood of a point and by building on the earlier concepts of subbasis and closure. We introduce and develop the new concepts of derived fuzzy set and relative fuzzy topology. We show that the collection of all fuzzy topologies on a fixed set is a complete lattice, give four characterizations of an open fuzzy set, characterize a neighborhood of a fuzzy set and establish two characterizations of a basis for a fuzzy topology. All of the contributions in this paper which are not referenced are original.

Fuzzy topological spaces are a very natural generalization of topological spaces in the following sense. As a result of the (1-1)-correspondence between the family of all subsets of a set X and the set of all characteristic functions which have domain X , a topology on X can be regarded as a family of characteristic functions with the usual set operations of $\subset$, $\cup$, $\cap$ and complementation replaced by the function operations of $\leq$, $\vee$, $\wedge$ and $1 - \mu_E$, respectively. A fuzzy

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