

NON-LINEAR QUANTUM FIELD THEORY*

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0. **Introduction.** This review is a greatly extended version of a set of lectures given at the Soliton conference in Tucson, Arizona.

It is written for mathematicians who have at least a casual acquaintance with systems that support soliton behavior, though one does not have to be a specialist to follow it. The lectures presume no knowledge of quantum field theory, despite the title. The idea was to present methods for investigating non-linear phenomena intrinsic to certain systems of physically interesting equations arising in quantum field theory, and so physics will play a muted role. Some of the material will involve familiar systems, e.g., the sine-Gordon equation, analyzed in an unfamiliar setting; for certain classical perfect systems are so stable they survive intact the process of second quantization.

The methods we discuss are new, this being a review of work done in the area by us and others in the last two years. Our main analytic tool is the functional, or Feynman path integral and approximations to it. Since we assume this is unfamiliar to most mathematicians, we have included a short course on the subject in Section 1, adequate to follow the subsequent development. All the quantum field theory, represented in the language of path integrals, that one needs is introduced where appropriate. There are many excellent texts available for those who wish to know more than we have space to present here. The bibliography contains a short list of relevant papers, which in turn contain references to the already extensive literature.

This is the first time in a century that the frontiers of physics and mathematics share a common ground. The most surprising consequences of nonlinear phenomena for modern physics are in the future, but already there are indications that they play a fundamental role in the field theory description of the sub-nuclear world.

1. **A short course on path integrals.** In the literature, path integrals, functional path integrals, and Feynman path integrals, are names used interchangeably. We will make no attempt at giving a rigorous justifi-

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