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# GOING-BETWEEN RINGS AND CONTRACTIONS OF SATURATED CHAINS OF PRIME IDEALS

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ABSTRACT. The concept of a going-between ring $A$ (that is, if $A \subseteq B$ are rings such that $B$ is integral over $A$, if $P \subset Q$ are prime ideals in $B$, and if there exists a prime ideal $p'$ in $A$ such that $P \cap A \subset p' \subset Q \cap A$, then there exists a prime ideal $P'$ in $B$ such that $P \subset P' \subset Q$) is introduced and a number of characterizations of such rings in terms of factor rings, quotient rings, and contractions of saturated chains of prime ideals are given. The relationship between such rings and catenary-like conditions on a ring is considered, and two additional characterizations of Noetherian going-between rings are given.

1. **Introduction.** All rings in this article are assumed to be commutative with non-zero identity element. The teminology is, in general, the same as that in [6].

This paper is concerned with two concepts which have recently been deeply investigated. First, somewhat analogous to GU-rings and GD-rings which have been studied in many papers, including [1, 4], we consider GB-rings (going-between rings (see (2.1))), characterize them in a number of ways, consider the special case of Noetherian GB-rings, and, finally, polynomial extensions of such rings are also considered. Second, the results are related to catenary-like conditions on a ring. This is due to the following result (3.8): A ring $A$ satisfies the c.c. (3.7.4) if and only if $A$ is catenary and a GB-ring. Now, rings which satisfy the c.c. have been investigated in many papers, including [5, 7, 8], and catenary rings were investigated in [9]. The results in this paper are thus in this line of reasearch, since they have to do with the other property (being a GB-ring) of rings which satisfy the c.c.

In § 2 we first define GB-rings (2.1), and then list four useful facts about when $A \subseteq B$ satisfy GB.

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