INVARIANT SETS AND THE HUKUHARA-KNESER PROPERTY FOR SYSTEMS OF PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

J. W. BEBERNES AND K. SCHMITT*

1. **Introduction.** During the past few years much work has been devoted to the problem of characterizing sets which are invariant with respect to a given ordinary differential equation. More recently the papers [2], [15] have addressed themselves to the same question for nonlinear parabolic differential equations.

The purpose of this paper is twofold. First we provide some extensions of invariance results (for parabolic equations) (sections 3 and 4) and secondly show that the assumptions which are sufficient for a given region to be invariant also yields existence of solutions of initial boundary value problems. We further show that the systems considered have the classical Hukuhara-Kneser property, i.e., the set of solutions of a given initial boundary value problem is a continuum in an appropriate function space; we thus provide an extension of a result of [5] to a large class of systems of parabolic differential equations.

Our invariance results were motivated by a result of [1], where certain geometric conditions were given to establish the solvability of two point boundary value problems for systems of second order ordinary differential equations; the type of result given there is the following: *Given a nonempty bounded open convex set such that the vector field defined by the nonlinear terms in the differential equation never points into the interior of the convex set, then for any two points in the convex set there exists a solution of the equation connecting the two points and which has values in that set.*

It is precisely these conditions that were adopted in [15] to show that they implied invariance for a system of parabolic equations. Under somewhat weaker assumptions than in [15] we not only prove invariance of that convex region but also demonstrate existence of solutions. Using some ideas suggested by [11] we further show that essentially the same type of result holds for convex sets with empty interior.

In order to establish the Hukuhara-Kneser property for systems of parabolic equations satisfying our conditions we rely on results and ideas about the structure of the set of fixed points of completely con-