ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 7, Number 2, Spring 1977

ON STRONG RIESZ AND STRONG GENERALIZED CESÀRO SUMMABILITY

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1. **Introduction.** We suppose throughtout that $\lambda = \{\lambda_n\}$ is a given sequence satisfying

$$(1) \qquad 0 = \lambda_0 < \lambda_1 < \cdots < \lambda_n \to \infty .$$

For $\kappa \geqq 0$, $\mu > 0$, $p = 0, 1, 2, \cdots$, and a given (real or complex) number s, we define the following means of an infinite (real or complex) series $\sum a_n$ (here, and throughout, $\sum$ means $\sum_0^\infty$ unless otherwise specified). The (R, λ, κ)-mean:

$$(2) \qquad R^\kappa(\tau) = \sum_{\lambda_\nu < \tau} (1 - \lambda_\nu/\tau)^\kappa a_\nu \qquad (\tau > 0);$$

the $[R, \lambda, \kappa + 1]_\mu$-mean:

$$(3) \qquad F^{\kappa+1}(\omega) = \omega^{-1} \int_0^\omega |R^\kappa(\tau) - s|^\mu \, d\tau \qquad (\omega > 0);$$

the (C, λ, p)-mean:

$$(4) \qquad \begin{aligned} t_n^0 &= s_n = \sum_{\nu=0}^n a_\nu, \\ t_n^p &= \sum_{\nu=0}^n (1 - \lambda_\nu/\lambda_{n+1}) \cdots (1 - \lambda_\nu/\lambda_{n+p}) a_\nu \qquad (n = 0, 1, \cdots); \end{aligned}$$

the $[C, \lambda, p + 1]_\mu$-mean:

$$(5) \qquad \sigma_m^{p+1} = \sum_{n=0}^m a_{mn} |t_n^p - s|^\mu \qquad (m = 0, 1, \cdots),$$

where $a_{mn} = (\lambda_{n+p+1} - \lambda_n) E_n^p / E_m^{p+1}$ $(0 \leqq n \leqq m)$, $a_{mn} = 0$ $(m > n)$, and $E_n^p = \lambda_{n+1} \cdots \lambda_{n+p}$ (with E_n^0 defined as 1).

Ordinary and strong Riesz summability (of real order) and ordinary, absolute, and strong generalized Cesàro summability (of integer

[1]This paper was written while the first and third authors were the recipients of research grants awarded by the National Research Council of Canada.

Received by the editors on May 23, 1975.